Controlling decoherence of a two-level-atom in a lossy cavity

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(Date: February 1, 2008)

By use of external periodic driving sources, we demonstrate the possibility of controlling the coherent as well as the decoherent dynamics of a two-level atom placed in a lossy cavity. The control of the coherent dynamics is elucidated for the phenomenon of *coherent destruction of tunneling* (CDT), i.e., the coherent dynamics of a driven two-level atom in a quantum superposition state can be brought practically to a complete standstill. We study this phenomenon for different initial preparations of the two-level atom. We then proceed to investigate the decoherence originating from the interaction of the two-level atom with a lossy cavity mode. The loss mechanism is described in terms of a microscopic model that couples the cavity mode to a bath of harmonic field modes. A suitably tuned external cw-laser field applied to the two-level atom slows down considerably the decoherence of the atom. We demonstrate the suppression of decoherence for two opposite initial preparations of the atomic state: a quantum superposition state as well as the ground state. These findings can be used to the effect of a proficient battling of decoherence in qubit manipulation processes.

I. INTRODUCTION

The idea of controlling the coherent dynamics of a quantum system by an external time-dependent force has found wide spread experimental and theoretical interest in many areas of physics (for reviews see [1–3]). It is, e.g., a commonly used tool to manipulate trapped atoms in quantum optics [4,5] as well as to control chemical reactions by a strong laser field [1–3]. In the context of quantum optics, it has been demonstrated experimentally [6] that a frequency-modulated excitation of a two-level atom by use of a microwave field driving transitions between two Rydberg Stark states of potassium significantly modifies the time evolution of the system. In the context of tunneling systems it has also been demonstrated that it is in principle possible to completely suppress the coherent tunneling of an initially localized wave packet in a double-well potential by an external, suitably designed time-periodic cw-perturbation (*coherent destruction of tunneling*) [7].

However, real quantum systems are always in contact with their environment. The coherent dynamics is then usually destroyed due to the influence of the large number of environmental degrees of freedom. Not only the phase of the quantum system is disturbed (decoherence) [8] but also energy exchange (dissipation) [9,10] takes place between the system under consideration and the environment. An example of such a system-bath interaction is the ensemble of electromagnetic field modes in a cavity, each of which is described as a quantum mechanical harmonic oscillator [4]. Each mode interacts with an atom trapped in the cavity. On the other side, the cavity modes themselves are also not isolated from the macroscopic environment; as such they are more realistically described as damped quantum harmonic oscillators. A topic of fundamental interest is the decay of quantum superpositions of states. In [11] it is shown how quantum optical nonclassical states are highly sensitive to dissipation stemming from a zero-temperature heat bath. Experimental works studying decoherence systematically are rare. In [12] the decoherence of mesoscopic superpositions of field states in the cavity has been investigated. In a recent work, Wineland and collaborators [13] demonstrate that the decoherence rate scales with the square of a quantity that describes the separation between two initial states. Moreover, Knight and co-workers [14] proposed an experimental scheme to probe the decoherence of a macroscopic object.

In this spirit, the question arises to which extent it is possible to control the dynamics of a quantum system in presence of decoherence and, moreover, whether the effect of decoherence can be minimized by an external time-dependent force, e.g., by a laser field [1,7,15–18,20]. To achieve this goal, various approaches have been undertaken in recent years. (i) It has been shown that the effect of coherent destruction of tunneling (see above) can be used to *slow down* the relaxation of a quantum system to its asymptotic equilibrium [16]. (ii) Moreover, a suitable tailored sequence of radio-frequency pulses ("quantum bang-bang" [17] or "parity kicks" [18]) that repeatedly flip the state of a two-level atom may be used to suppress decoherence. (iii) The cavity-induced spontaneous emission of a two-level

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atom can be manipulated by a strong rf field which couples to the cavity mode [19]. (iv) The manipulation of the system-bath interaction by a fast frequency modulation also results in slowing down decoherence and relaxation [20].

The objective of this work is to study the influence of a time-periodic driving field on the dynamics of a two-level atom. In the first part of this work (section II), we deal with the objective to "freeze" a coherent dynamics, i.e., we shall employ an effect known as coherent destruction of tunneling. Most importantly, we investigate this freezing phenomenon from the viewpoint of its dependence on different initial preparations.

In the second part of this work (section III), we do not elaborate further on the effect of coherent destruction tunneling, but instead investigate the control of decoherence of a two-level atom placed in a lossy cavity. Our model consists of a two-state system which is coupled to a time-dependent periodic field. The driven two-state system interacts furthermore with one mode of the cavity having the frequency Ω. This mode is itself damped by the coupling to a bath of harmonic oscillators (*lossy cavity*). It is known [21] that a Hamiltonian consisting of (1) a system part, (2) a harmonic oscillator with frequency Ω that is being coupled to the system, and (3) a bath of harmonic oscillators which are coupled to this very single harmonic oscillator can be mapped onto a Hamiltonian composed of the system part coupled to a harmonic bath with an effective spectral density. This effective spectral density possesses a Lorentzian-shaped peak at Ω. The completely isolated atom (no driving, no cavity) evolves in time in a coherent way according to the Schrödinger equation. It is this dynamics which we want to preserve and protect as far as possible from the decoherent influence of the environment. Our major finding is that a cw-control field can indeed be used to (i) reduce decoherence and (ii) to restore to some extent the unperturbed, non-dissipative time-evolution.

II. THE DRIVEN TWO-LEVEL ATOM

A. Floquet Formalism

To start we consider a Hamiltonian describing a two-level atom with the ground state $|1\rangle$ and an excited state $|2\rangle$. The energy levels are separated by the energy $\hbar\Delta_0$. The atom with the transition dipole moment μ is driven within the long wavelength approximation by an external, time-dependent laser field of the form $\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega_{\mathrm{L}} t)$ with frequency ω_{L} and amplitude $\mathcal{E}_0$, yielding the driven quantum system

$$H(t) = -\frac{\hbar}{2}[\Delta_0 \hat{\sigma}_z + s(t)\hat{\sigma}_x]\,. \tag{1}$$

Here, the matrices $\hat{\sigma}_i$, $i = x, y, z$ denote the Pauli spin matrices. The part involving $s(t) = s \cos(\omega_{\mathrm{L}} t)$ with $s = 2\mu\mathcal{E}_0/\hbar$ presents the time-dependent driving which couples to the transition dipole moment μ of the atom. Note that within this scaling the amplitude s possesses the dimension of a frequency. The driven time evolution of the populations of the energy levels exhibits an oscillatory behaviour. For an initial preparation of the atom in the ground state and for a resonant driving, i. e. $\omega_{\mathrm{L}} = \Delta_0$, and with s not large we can invoke the rotating wave approximation. The population of each state then oscillates between 0 and 1 with the Rabi frequency $\Omega_{\mathrm{R}} = s/2$. Because the Hamiltonian (1) is periodic in time with the period $\mathcal{T} = 2\pi/\omega_{\mathrm{L}}$, i.e., $H(t + \mathcal{T}) = H(t)$, we next apply for the general case away from resonance the Floquet formalism [1]. The time-dependent Schrödinger equation may be written as

$$\{H(t) - \mathrm{i}\hbar\partial/\partial t\}|\psi(t)\rangle = 0. \tag{2}$$

According to the Floquet theorem, there exist solutions to eq. (2) of the form

$$|\Psi_\alpha(t)\rangle = \exp(-\mathrm{i}\varepsilon_\alpha t/\hbar)|\Phi_\alpha(t)\rangle\,, \tag{3}$$

with $\alpha = 1, 2$. The periodic function $|\Phi_\alpha(t)\rangle$ are termed the Floquet modes and these obey

$$|\Phi_\alpha(t + \mathcal{T})\rangle = |\Phi_\alpha(t)\rangle\,. \tag{4}$$

Here, ε_α is the so called Floquet characteristic exponent or the *quasienergy*, which is real-valued and unique up to multiples of $\hbar\omega_{\mathrm{L}}$. Upon substituting eq. (3) into the Schrödinger equation (2) one obtains the eigenvalue equation for the quasienergy ε_α

$$\mathcal{H}(t)|\Phi_\alpha(t)\rangle = \varepsilon_\alpha|\Phi_\alpha(t)\rangle \tag{5}$$

with the Hermitian operator

$$\mathcal{H}(t) \equiv H(t) - \mathrm{i}\hbar\partial/\partial t\,. \tag{6}$$

We stress that the Floquet modes

$$|\Phi_{\alpha'}(t)\rangle = |\Phi_\alpha(t)\rangle \exp(\mathrm{i}n\omega_\mathrm{L}t) \equiv |\Phi_{\alpha n}(t)\rangle \tag{7}$$

with n being an integer number $n = 0, \pm 1, \pm 2, \ldots$ yield equivalent solutions to eq. (3) but with the shifted quasienergy

$$\varepsilon_\alpha \to \varepsilon_{\alpha'} = \varepsilon_\alpha + n\hbar\omega_\mathrm{L} \equiv \varepsilon_{\alpha n}\,. \tag{8}$$

Therefore, the index α corresponds to a whole class of solutions indexed by $\alpha' = (\alpha, n)$. The eigenvalues $\{\varepsilon_\alpha\}$ can thus be mapped into a first Brillouin zone obeying $-\hbar\omega_\mathrm{L}/2 \le \varepsilon < \hbar\omega_\mathrm{L}/2$. It is clear that for our choice of the external driving force, i.e. $s(t) = s\cos\omega_\mathrm{L}t$ the quasienergies are functions of the driving amplitude s and the driving frequency ω_L. For adiabatically vanishing external driving they merge into the eigenvalues of the time-independent part of the Hamiltonian (1), i.e.,

$$\varepsilon_{\alpha n}(s, \omega_\mathrm{L}) \xrightarrow{s\to 0} \mp\hbar\Delta_0/2 + n\hbar\omega_\mathrm{L}\,, \tag{9}$$

where the negative (positive) sign corresponds to $\alpha = 1\,(\alpha = 2)$. The Floquet modes, correspondingly, turn into the eigenfunctions $|\alpha\rangle$ multiplied by an additional phase factor, i.e.,

$$|\Phi_{\alpha n}(t)\rangle \xrightarrow{s\to 0} |\alpha\rangle \exp(\mathrm{i}\omega_\mathrm{L}nt)\,. \tag{10}$$

For a finite driving strength $s \neq 0$, the determination of the quasienergies ε_α requires the use of numerical methods. The interested reader is referred in this context to the literature [22,23]. However, we here state without proof that in the high-frequency regime $\Delta_0 \ll \max[\omega_\mathrm{L}, (s\,\omega_\mathrm{L})^{1/2}]$ the difference between the two quasienergies is given by [15]

$$\varepsilon_{2,-1} - \varepsilon_{1,1} = \hbar\Delta_0 J_0(s/\omega_\mathrm{L})\,, \tag{11}$$

where J_0 denotes the zeroth-order Bessel function of the first kind.

B. Freezing the coherent dynamics of a driven two-level system

Eq. (11) implies a most interesting consequence for a driven two-level system [15]: If one chooses the driving parameter s and ω_L in such a way that the argument of the Bessel function is at a zero of the Bessel function, the splitting between the quasienergy vanishes. Possible transitions between the Floquet states are then at most induced by the remaining periodic time-dependent parts of the corresponding Floquet modes $|\Phi_\alpha(t)\rangle$. This effect has been discovered in the context of tunneling systems. There, a wave packet being an equally weighted superposition of the symmetric and antisymmetric ground state is initially localized at one side of a double-well potential. By applying an external suitably tailored periodic field, the wave packet can be stabilized and can be prevented from coherently tunneling back and forth between the two wells, i.e., one finds *coherent destruction of tunneling* (CDT) [1,7]. We emphasize here that the crossing of two tunneling related quasienergy levels yields a *necessary* (but not sufficient) criterion for the suppression of coherent tunneling [15].

The challenge we want to address next is as follows: How does the driven dynamics of a two level atom that is being prepared in some arbitrary initial state evolve when the corresponding condition for the parameters obey the CDT-condition (11)? The system dynamics can be described by its density operator $\hat\rho(t)$, which is a 2×2-matrix, i.e.,

$$\hat\rho(t) = \hat I/2 + \sum_{i=x,y,z} \sigma_i(t)\hat\sigma_i/2\,, \tag{12}$$

where the expectation values $\sigma_i(t) := \mathrm{Tr}\{\hat\rho(t)\hat\sigma_i\}, i = x, y, z$ are the dynamical quantities of interest. $\hat I$ denotes the unit matrix and $\sigma_x(t)$ and $\sigma_y(t)$ are related to the coherences (the off-diagonal elements) of $\hat\rho(t)$ while $\sigma_z(t)$ is the population difference between the two energy eigenstates $|\alpha\rangle$. This implies that the state of the quantum system at time t is fully determined by the knowledge of the three expectation values $\sigma_i(t)$.

To determine the state of the driven two-level system at time t, we consider the Heisenberg equation of motion for the density matrix. Using the commutation relations for the $\hat\sigma_i$, we arrive at the equation of motion for the expectation values $\sigma_i(t)$ in (12), i.e.,

$$\dot{\sigma}_x(t) = -\Delta_0 \sigma_y(t)\,,$$
$$\dot{\sigma}_y(t) = \Delta_0 \sigma_x(t) - s(t)\sigma_z(t)\,,$$
$$\dot{\sigma}_z(t) = s(t)\sigma_y(t)\,. \tag{13}$$

To study the dependence of the effect of coherent destruction of tunneling on the initial preparation we first choose as initial state an equally weighted coherent superposition of the two unperturbed energy eigenstates, i.e.,

$$|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)\,, \tag{14}$$

corresponding to $\sigma_x(t=0) = 1, \sigma_y(t=0) = \sigma_z(t=0) = 0$. We solve the set of coupled differential equations (13) numerically by a standard fourth order Runge-Kutta integration algorithm with adaptive step-size control. In Fig. 1a, the time-dependence of the three expectation values is depicted. The driving parameters are chosen such that the condition (11) is fulfilled: in doing so we use $\omega_{\mathrm{L}} = 50\Delta_0$ and $s = 120.241...\Delta_0$. Surprisingly *all three* expectation values $\sigma_i(t)$ can be brought simultaneously to an almost perfect standstill !

Next, we choose the *ground state* as initial state, i.e., we use $|\Psi(t=0)\rangle = |1\rangle$. This corresponds to $\sigma_x(t=0) = \sigma_y(t=0) = 0, \sigma_z(t=0) = 1$. The result is depicted in Fig. 1b. Applying to the so prepared two-level system a laser field obeying the CDT-condition (11), we find that the y-component $\sigma_y(t)$ and the z-component $\sigma_z(t)$ exhibit strong oscillations. This oscillations follow from the numerically evaluated Floquet theory for the driven two-level system, and are *not* described by the Rabi-oscillations as predicted from a rotating wave approximation; this latter approximation is strongly violated for our chosen set of driving parameters. In contrast, $\sigma_x(t)$ can be stabilized around the initial value of zero. This finding is in accordance with the CDT-phenomenon: it reflects the fact that the corresponding two equally weighted ("left" and "right") localized parts of the ground state wave function of a double-well potential, as represented within a localized representation, can each be stabilized too.

This CDT-effect opens the doorway to manipulate the influence of an environment on a quantum system. It is known [16] that the coherent destruction of tunneling survives to some extent in presence of a coupling to the environment. Certainly, the system will relax in presence of an environment; however, as it is demonstrated in [16], the relaxation process can considerably be slowed down in the presence of a CDT-field.

In view of using differing initial preparations, the following remark should be made. From the viewpoint of stabilizing the state of a qubit (characterized by a quantum mechanical two-level system) in a quantum information processor [24], it is of foremost interest to stabilize the coherent superposition of two states of the qubit. Thus, our first choice (14) is of relevance in the context of the possibility for quantum computing. Moreover, fundamental questions concerning the decoherence of superposition states arise for the physics that occurs when one crosses the interface between the classical and quantum world, and vice versa [8,13].

III. CONTROL OF DECOHERENCE FOR A TWO-LEVEL ATOM

In this section we shall study the influence of an applied cw-control field for reducing decoherence of a two-level atom placed in a lossy cavity.

A. Driven two-level atom in a lossy cavity

To start we consider a two-level atom in a dissipative environment, e.g., a lossy cavity wherein the leakage of photons damps the radiation field. Additionally, the atom may be manipulated by a time-dependent external field like a laser beam. In our model, the driven two-level atom is represented by the Hamiltonian (1). It is coupled to one mode of the cavity which is described by one harmonic oscillator with frequency Ω, characterized by the annihilation and creation operators $\hat{B}$ and $\hat{B}^\dagger$ which fulfill the usual commutation relations for bosonic field operators. The coupling constant is denoted by g and has the dimension of a frequency. This cavity mode is damped by a bilinear coupling to a bath of harmonic oscillators of frequencies ω_i. They are similarly described by bosonic annihilation and creation operators $\hat{b}_i$ and $\hat{b}_i^\dagger$. The coupling constants of the cavity mode to the harmonic bath are given by κ_i and have the dimension of a frequency. The total system-bath Hamiltonian is therefore written as

$$H(t) = -\frac{\hbar}{2}[\Delta_0 \hat{\sigma}_z + s(t)\hat{\sigma}_x]$$
$$+ \hbar\Omega(\hat{B}^\dagger \hat{B} + \frac{1}{2}) + \hbar g(\hat{B}^\dagger + \hat{B})\hat{\sigma}_x$$

$$+\sum_{i=1}^{N}\hbar\omega_i(\hat{b}_i^\dagger\hat{b}_i+\frac{1}{2})+\hbar(\hat{B}^\dagger+\hat{B})\sum_{i=1}^{N}\kappa_i(\hat{b}_i^\dagger+\hat{b}_i)\,. \tag{15}$$

The influence of the bath on the two-level atom plus cavity mode is fully characterized by the spectral density

$$J(\omega)=2\pi\sum_{i=1}^{N}\kappa_i^2\,\delta(\omega-\omega_i). \tag{16}$$

We let the number of bath modes going to infinity ($N\to\infty$) and choose an Ohmic spectral density for the bath oscillators with an exponential cut-off at some large frequency $\omega_c \gg \Delta_0,\omega_{\rm L},\Omega$, i.e.,

$$J(\omega)=\frac{2\Gamma}{\Omega}\omega\exp(-\omega/\omega_c)\,, \tag{17}$$

Here, we have introduced the damping constant Γ which is related to the quality factor of the cavity. Since the cavity mode as well as the bath oscillators are described by harmonic oscillators, we follow the approach in [21] and map the Hamiltonian (15) onto a Hamiltonian where the central system, i.e., the two-level atom, is now bilinearly coupled to a bath of *mutually non-interacting* harmonic oscillators with an effective spectral density $J_{\rm eff}(\omega)$. Upon letting the cut-off frequency going to infinity, i.e., $\omega_c\to\infty$, this effective spectral density emerges as

$$J_{\rm eff}(\omega)=\frac{16\Gamma}{\Omega}\frac{g^2\omega\Omega^2}{(\Omega^2-\omega^2)^2+4\omega^2\Gamma^2}\,. \tag{18}$$

For small frequencies ω, it increases linearly like in the original Ohmic spectral density $J(\omega)$. However, it has a Lorentzian shaped peak at $\omega=\Omega$ with a line width $\Gamma<\Omega$.

In the following section, we make extensive use of the bath autocorrelation function $\mathcal{M}(t)=\mathcal{M}'(t)+{\rm i}\mathcal{M}''(t)$, which is obtained in terms of the effective spectral density $J_{\rm eff}(\omega)$, i. e.

$$\mathcal{M}(t)=\frac{1}{\pi}\int_0^\infty d\omega J_{\rm eff}(\omega)\left[\coth\left(\frac{\hbar\omega}{2k_{\rm B}T}\right)\cos(\omega t)-i\sin(\omega t)\right]\,. \tag{19}$$

All our further considerations treat the case when the bath is at zero temperature, i.e. $T=0$. In this limit, and for our choice of the effective spectral density (18), we obtain for the real and imaginary part, respectively, the analytical results

$$\begin{aligned}\mathcal{M}'(t)&=\frac{16\Gamma}{\pi\Omega}g^2\Omega^2\big[\frac{\pi}{4}\frac{1}{\Gamma\sqrt{\Omega^2-\Gamma^2}}e^{-\Gamma t}\,\cos(\sqrt{\Omega^2-\Gamma^2}t)-\int_0^\infty dy\frac{ye^{-yt}}{(y^2+\Omega^2)^2-4y^2\Gamma^2}\big]\,,\\ \mathcal{M}''(t)&=-4g^2\frac{\Omega}{\sqrt{\Omega^2-\Gamma^2}}e^{-\Gamma t}\sin(\sqrt{\Omega^2-\Gamma^2}t)\,.\end{aligned} \tag{20}$$

The quantity of interest is the reduced density matrix for the two-level system which we denote – just as in the undamped case – by $\hat{\rho}(t)$. It follows by tracing over the bath degrees of freedom in the full density operator $\hat{W}(t)$ which corresponds to the system-plus-bath Hamiltonian (15), i.e., $\hat{\rho}(t)={\rm tr_B}\hat{\rm W}({\rm t})$. Like in the deterministic case in eq. (12), $\hat{\rho}(t)$ is fully characterized by the expectation values $\sigma_i(t), i=x,y,z$. We shall determine their corresponding equations of motions next.

B. Bloch - Redfield Formalism

To deal with quantum dissipative systems, several techniques have been developed [1,9,10,22]. A very efficient numerical algorithm for general quantum system with a discrete eigenvalue spectrum has been developed by Makri and Makarov within the real-time path-integral formalism [25]. It has also been applied to spatially continuous tunneling systems in presence of driving [26]. Moreover, the real-time path-integral formalism has extensively been used to describe a moderate-to-strong (!) two-level system-bath interaction [1,9,21,27]. Recently, the former scheme has been generalized to describe multi-level, driven vibrational and tunneling dynamics in [28]. At weak system-bath coupling the Nakajima-Zwanzig projector operator theory [29] provides a powerful tool to describe the corresponding reduced density matrix dynamics.

For our quantum optical problem at hand, the suitable method of choice in presence of a physically realistic weak system-bath coupling is the projection operator technique: it yields in Born approximation the generalized master

equation. It can be simplified further without loss of accuracy in leading order in the (weak) coupling strength g by invoking the Markovian approximation [32]. For a strong harmonic driving this objective was formally (only) developed a long time ago by Argyres and Kelley [30]. Following the reasoning in [30] (see in this context also [31]) we recently have derived for this case of a driven spin-boson problem with an *arbitrary* control field the explicit set of coupled, Bloch-Redfield type equations [32]

$$\begin{aligned}\dot{\sigma}_x(t) &= -\Delta_0\sigma_y(t)\,,\\ \dot{\sigma}_y(t) &= \Delta_0\sigma_x(t) - s(t)\sigma_z(t) - \Gamma_1(t)\sigma_y(t) - \Gamma_2(t)\sigma_x(t) - A_y(t)\,,\\ \dot{\sigma}_z(t) &= s(t)\sigma_y(t) - \Gamma_1(t)\sigma_z(t) - \Gamma_3(t)\sigma_x(t) - A_z(t)\,.\end{aligned} \tag{21}$$

The time-dependent rates $\Gamma_i(t) = \int_0^t dt'\mathcal{M}'(t-t')b_i(t,t')$, together with the inhomogeneities $A_y(t) = \mathrm{Re}F(t)$, $A_z(t) = \mathrm{Im}F(t)$, with $F(t) = (1/2)\int_0^t dt'\mathcal{M}''(t-t')[u^2(t,t') - v^2(t,t')]$ determine the dissipative action of the thermal bath on the two-level atom. The functions $\mathcal{M}'$ and $\mathcal{M}''$ denote the real part and imaginary part, respectively, of the correlation function $\mathcal{M}$ given in eqns. (20). The quantities $u(t,t') = \langle 1|\hat{U}(t,t')|1\rangle + \langle 2|\hat{U}(t,t')|1\rangle$ and $v(t,t') = \langle 1|\hat{U}(t,t')|2\rangle + \langle 2|\hat{U}(t,t')|2\rangle$ are sums of matrix elements of the time evolution operator $\hat{U}(t,t')$ of the *isolated* (i.e., $g = 0$) driven two-level system. The functions b_i read $b_1 = \mathrm{Re}uv^*$, $b_2 = -(1/2)\,\mathrm{Im}(u^2 - v^2)$, and $b_3 = (1/2)\,\mathrm{Re}(u^2 - v^2)$. Note that this set of equations is valid in the parameter regime $g \ll \Delta_0/2$. One can demonstrate that for the undriven case, i.e., $s = 0$, the analytic solution of eq. (21) in first order in g reproduces the analytical path integral weak-damping results in Refs. [9,33].

C. Controlling the decoherence of a quantum superposition of states

The idea of controlling the decohering influence of the environment on a quantum system by an external time-dependent field is demonstrated for the case of the two-level atom which is initially prepared in an equally weighted superposition of the two energy eigenstates given by $\sigma_x(t=0) = 1, \sigma_y(t=0) = \sigma_z(t=0) = 0$. In doing so, we consider four different situations: (1) first, we look at the isolated two-level atom dynamics without driving and without coupling the atom to the lossy cavity mode. This case corresponds to setting $s = 0$ and $g = 0$. Case (2) is devoted to the driven two-level dynamics. We switch on a coherent driving cw-field but keep the system isolated from the bath, i.e. $s \neq 0$ and $g = 0$. In case (3) we investigate how the undriven system dynamics relaxes in presence of a dissipative coupling to the bath. We therefore set $s = 0$ and $g \neq 0, \Gamma \neq 0$. Finally, we demonstrate with case (4) how this decoherent dynamics can be manipulated with the help of an externally applied time-dependent control field and set $s \neq 0, g \neq 0$ and $\Gamma \neq 0$.

In order to preserve the coherent evolution of the two-level atom and to protect it as far as possible from the decoherent influence of the environment, we choose the following control scheme: Guided by the physics of a rotating wave approximation for the driven system that most closely retains the unperturbed dynamics of an initial superposition state (14) we choose the following parameters: The frequency and the amplitude of the driving field are taken to be *in resonance* with the level spacing of the two-level system, i.e., $\omega_\mathrm{L} = \Delta_0$ and $s = \Delta_0$ which corresponds to a moderately strong driving strength. This choice implies for the ratio of the corresponding Rabi-frequency and driving strength the value 0.5. This indicates that the rotating wave approximation should be used already with caution. Note that under the CDT-condition in (11), the field strength would assume an even larger value of $s = 2.4048\Delta_0$. For the strength of the coupling between the two-level atom and the cavity mode we assume $g = 0.05\Delta_0$. This value is consistent with the range of validity of the Bloch-Redfield formalism in Born approximation (see above). The dissipative system-bath mechanism is specified as follows: the frequency of the cavity mode is chosen to be in resonance as well, i.e, $\Omega = \Delta_0$. By doing so, we in essence maximize the influence of the bath. For the line width of the cavity mode we set $\Gamma = 0.1\Delta_0$. This rather large value mimics (on purpose) an extreme situation because the line width in most realistic situations is in general much smaller: Nevertheless, such smaller values would intensify our appealing finding of a driving-induced, enhanced *recovery of coherence* even more. Moreover, the temperature is always set to $T = 0$.

Our novel results are depicted in the Fig. 2 (a)-(c) and the Fig. 3 (a)-(c). Fig. 2a depicts the time-evolution of the x-component $\sigma_x(t)$. The isolated two-level dynamics (dashed line) shows coherent oscillations between -1 and 1 at the frequency of the level spacing Δ_0. On top of this line one finds (barely visible dotted line) the results for the driven two-level dynamics. This good agreement follows also from the corresponding rotating wave approximation, yielding for this preparation just the undriven result. The decoherence in presence of a finite bath coupling ($g = 0.05\Delta_0, \Gamma = 0.1\Delta_0$), see the dashed-dotted line, yields an oscillatory decay towards equilibrium $\sigma_x(t \to \infty) = 0$, whose envelope is made visible by the connecting solid line. Next we switch on the cw-laser control field. As a main result we find that the decoherence becomes considerably slowed down − following closely the isolated driven dynamics. This enhanced recovery of coherence for the dissipative *driven* dynamics is made visible to the eye by the connecting weakly decaying

and oscillating envelope. This surprising result is rooted in the following facts: The dissipative, non-driven dynamics experiences a most effective dissipation. This is due to the resonant coupling at $\Omega = \Delta_0$ of the two-level atom to bath with the effective spectral density in (18) which peaks at $\omega = \Omega$. In contrast, the strong driving now dresses this level spacing, and moves it out of resonance with the lossy cavity mode. This results in a considerable slow down of driven decoherence for $\sigma_x(t)$.

The decoherent dynamics of the y-component $\sigma_y(t)$ is qualitatively similar to $\sigma_x(t)$. It is depicted in Fig. 2b for the same choice of parameters.

The population difference $\sigma_z(t)$ is shown in Fig. 2c. For the isolated two-level dynamics, $\sigma_z(t)$ remains constant at zero (dashed line) since the system is in an equally weighted superposition of two eigenstates, yielding an obvious zero population difference. In presence of the cw-laser control field, the driven dynamics (dotted line) yields a finite oscillation of population difference. This deviation from zero also reflects the deviation from the corresponding rotating wave solution (being identically zero for this preparation). Nevertheless, this driven dynamics still exhibits an approximate periodicity that closely coincides with the Rabi-value $\Omega_R = \Delta_0/2$.

The undriven, dissipative relaxation to equilibrium (dashed-dotted line) proceeds with temperature $T = 0$ almost completely towards the ground state with corresponding maximal population difference $\sigma_z(t \to \infty) \approx 1$. Due to the coupling to the cavity mode performing zero point oscillations, the value of 1 is not fully reached. The driven, dissipative relaxation (solid line) to the time periodic asymptotic state exhibits oscillations around zero – following initially (up to $\Delta_0 t \approx 50$) closely the driven coherent dynamics. In virtue of Floquet theory for the long-time limit of the time-periodic generalized Bloch-Redfield equations in (11), this asymptotic periodicity matches in the lon-time limit the frequency of driving, i.e., $\omega_L = \Delta_0$ (not depicted).

D. Controlling the decoherence from the atom ground state

To answer the question whether the proposed control scheme works as well in the opposite limit of an initial state which is an eigenstate we next choose the ground state as the initial preparation, i.e., we use $\sigma_x(t = 0) = \sigma_y(t = 0) = 0, \sigma_z(t = 0) = 1$. The remaining parameters are taken to be the same as in the previous subsection III C.

Fig. 3a shows the decoherent dynamics for $\sigma_x(t)$. Since the chosen initial state is an eigenstate of the isolated two-level system no dynamics is exhibited (note the filled squares on the line at zero in the figure). This situation remains unaltered in presence of a dissipative coupling of the quantum system, as indicated by the asterisks on the line at zero. At zero temperature the system at weak dissipation remains essentially in its ground state. Upon switching on the driving with no coupling to the lossy cavity present, the driven two-level dynamics exhibits a Rabi-like quasiperiodic, oscillatory behaviour (dotted line). This nonperiodic behavior is rooted in the deviation of the full Floquet dynamics from a rotating wave prediction. With our strong driving strength we *a priori* cannot expect good agreement with the corresponding rotating wave approximation. The coupling to the lossy cavity mode damps this quasiperiodic behaviour, following for short times the driven isolated dynamics (see solid line), before settling down to asymptotic, long-time oscillations at the frequency of driving $\omega_{\rm L} = \Delta_0$ with a finite, but strongly reduced amplitude (not depicted).

The decoherent dynamics of the y-component $\sigma_y(t)$ is again qualitatively similar to that of $\sigma_x(t)$. It is presented in Fig. 3b for the same set of coupling and driving parameters.

Finally, the time evolution of the population difference $\sigma_z(t)$ is depicted with Fig. 3c. Clearly, the isolated dynamics from a prepared initial ground state remains constant at $\sigma_z(t) = 1$ (filled squares). The driven dynamics of the two-level system exhibits strong non-detuned Rabi oscillations at frequency $\Omega_R = \Delta_0/2$ between -1 and 1 (dotted line). In this case the rotatating wave prediction (not depicted) actually yields surprisingly good qualitative agreement with the exact dynamics.

The case of no driving ($s = 0$) but with a coupling to the bath ($g = 0.05\Delta_0, \Gamma = 0.1\Delta_0$) shows again a trivial dynamics. It relaxes in this case of weak dissipation with a small relaxation rate towards a slightly reduced constant value close to 1 (indicated by the asterisks).

The case with resonant driving ($s = \Delta_0, \omega_{\rm L} = \Delta_0$) switched on and simultaneous coupling to the lossy cavity mode (with $g = 0.05\Delta_0, \Gamma = 0.1\Delta_0$) exhibits damped Rabi-oscillations (solid line); it eventually settles down in the asymptotic long-time limit to periodic asymptotic oscillations at twice the Rabi frequency and amplitude smaller than 1 (not depicted).

IV. CONCLUSIONS

In this work we have investigated the possibility to control the time-evolution of a two-level atom by time-dependent external, periodic control forces. We have demonstrated that the coherent dynamics of the system can be brought

to an almost perfect standstill by choosing the ratio of driving amplitude s and driving frequency $\omega_{\rm L}$ at a zero of the Bessel function $J_0(s/\omega_{\rm L})$ (*coherent destruction of tunneling*). For an initially prepared quantum superposition of states all three components $\sigma_i, i = x, y, z$ and therefore the entire density matrix $\hat{\rho}$ can be locked simultaneously. For the initially prepared ground-state, the x-component σ_x can be stabilized; the other two components σ_y and σ_z, however, depict strong (non-Rabi) oscillations.

In presence of decoherence in a lossy cavity we illustrate that the atomic states can be dressed by a time-dependent force which moves the atom and the cavity mode out of resonance. As a consequence, decoherence becomes strongly suppressed. We have illustrated this effect for two different initial preparations of the atom: (i) for a quantum superposition of states we show that the decoherence can be suppressed efficiently. (ii) The second preparation uses the ground-state wave function of the isolated system. In that case the decoherence may also be slowed down, but the decohering dynamics never approaches again the initial state.

These findings put the idea across that the method can be used to bring back the state of the atom close to its initial preparation. For the case (i) of a superposition state as initial state the decoherent dynamics of the x- and y-component σ_x, σ_y are similar to the undriven dynamics of the isolated two-level system (qubit). Even the z-component σ_z of the driven dissipative dynamics matches at distinct instants of time the undriven non-dissipative dynamics. For the second case (ii) of the ground state as initial state this idea, however, seems to fail for the z-component σ_z.

To summarize, our proposed scheme for controlling the coherent and decoherent dynamics of a two-level atom works very well for initially prepared quantum superpositions of states. This presents good news for the manipulations of quantum bits (two-level systems) being in a superposition of states. It is this very feature which makes quantum computation interesting and superior to classical computation.

ACKNOWLEDGEMENT

This work has been supported by the Deutsche Forschungsgemeinschaft within the Schwerpunktsprogramm *Zeitabängige Phänomene und Methoden in Quantensystemen der Physik und Chemie*, HA1517/14-3 (L.H., I.G., P.H.), within the Schwerpunktsprogramm *Quanten-Informationsverarbeitung* HA1517/19-1, (M.T., P.H.) and in part by the Sonderforschungsbereich 486 of the Deutsche Forschungsgemeinschaft (I.G., P.H.).

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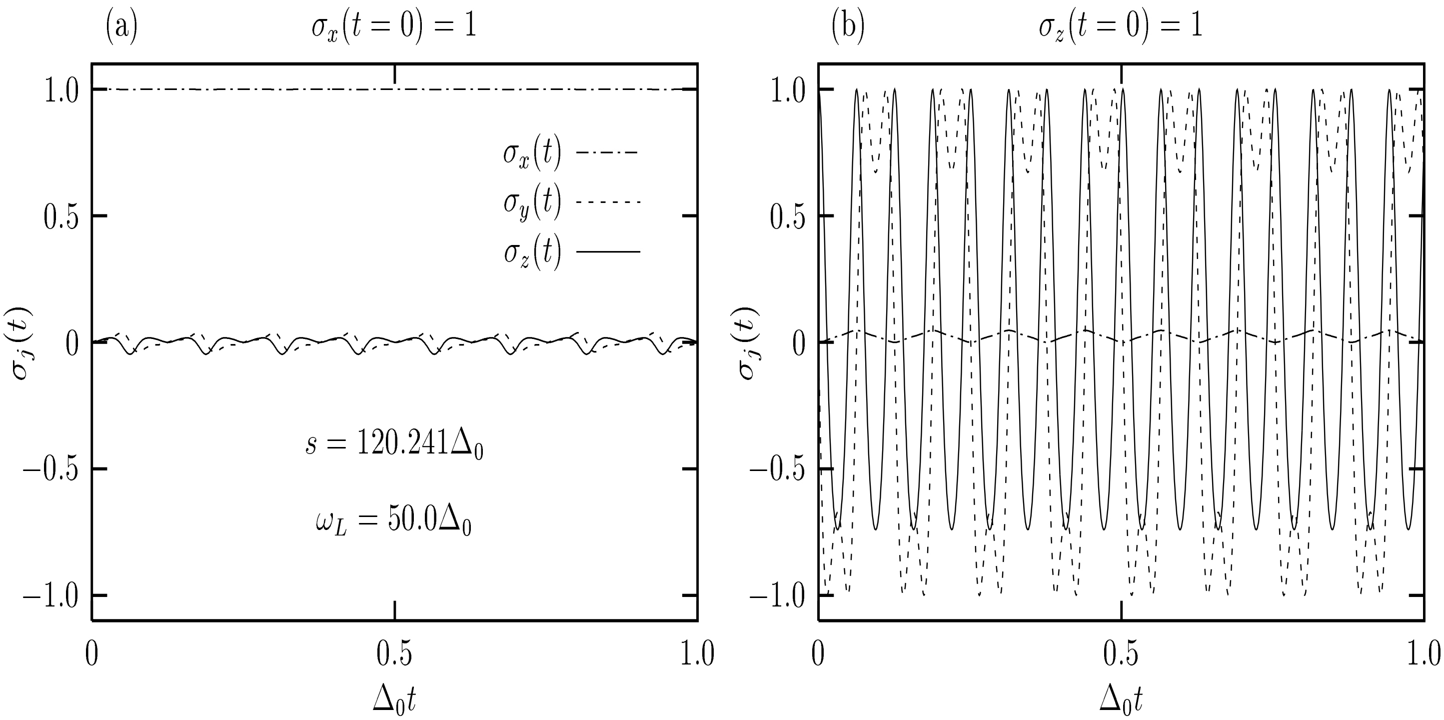

FIG. 1. Fig. 1a: Time-dependence of the expectation values $\sigma_j(t), j = x, y, z$, determining the dynamics of the full density matrix of the two-level atom according to (12,13). The driving parameters are chosen such that the condition (11) for coherent destruction of tunneling is fulfilled, i.e., the driving amplitude is set to $s = 120.241\Delta_0$ and the driving frequency $\omega_{\rm L} = 50\Delta_0$. The two-level system is prepared in an equally weighted superposition of the two energy eigenstate, i.e., $\sigma_x(t = 0) = 1, \sigma_y(t = 0) = \sigma_z(t = 0) = 0$. Fig. 1b: The same as in Fig. 1a but for an initial state preparation being the ground state of the two-level system, i.e., $\sigma_x(t = 0) = \sigma_y(t = 0) = 0, \sigma_z(t = 0) = 1$.

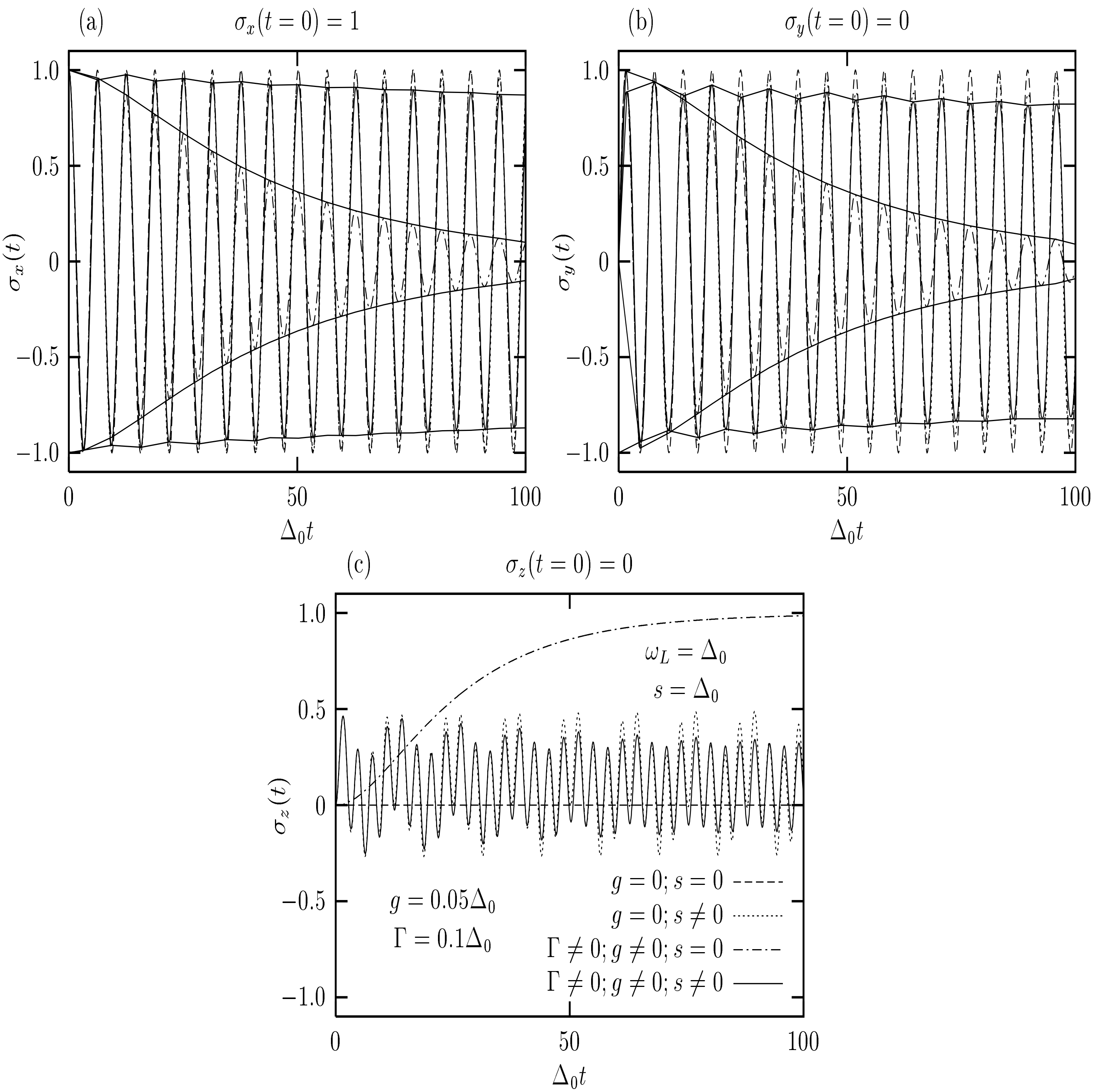

FIG. 2. Time-dependence of the expectation values $\sigma_x(t)$ (Fig. 2a), $\sigma_y(t)$ (Fig. 2b) and $\sigma_z(t)$ (Fig. 2c) for the two-level atom initially prepared in an equally weighted superposition of the energy eigenstates, i.e. $\sigma_x(t=0)=1, \sigma_y(t=0)=\sigma_z(t=0)=0$. Shown are four cases: (1) no driving ($s=0$), zero system-cavity mode coupling ($g=0$) (dashed line), (2) with resonant driving ($s=\Delta_0, \omega_{\rm L}=\Delta_0$), but zero coupling ($g=0$)(dotted line), (3) zero driving ($s=0$), but with finite coupling ($g=0.05\Delta_0, \Gamma=0.1\Delta_0$) (dashed-dotted line) and (4) with resonant driving ($s=\Delta_0, \omega_{\rm L}=\Delta_0$) and with coupling ($g=0.05\Delta_0, \Gamma=0.1\Delta_0$) (full line). The temperature is chosen to be $T=0$ and the cavity-mode frequency is set to $\Omega=\Delta_0$. As a guide for the eye, we mark the envelope of the decaying oscillations by solid lines in Fig. 2a and Fig. 2b.

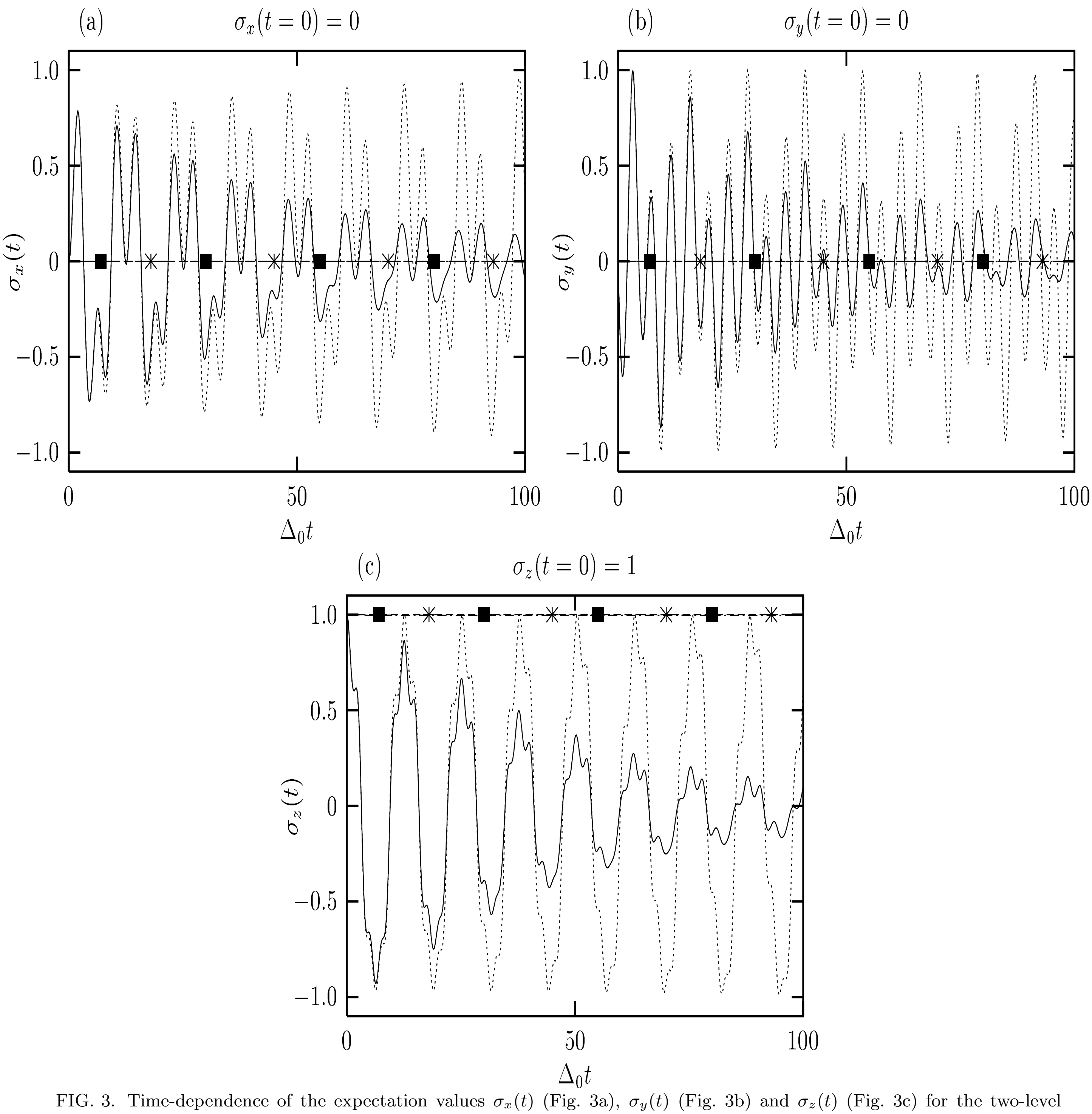

FIG. 3. Time-dependence of the expectation values $\sigma_x(t)$ (Fig. 3a), $\sigma_y(t)$ (Fig. 3b) and $\sigma_z(t)$ (Fig. 3c) for the two-level atom initially prepared in the ground-state, i.e., $\sigma_x(t=0)=\sigma_y(t=0)=0, \sigma_z(t=0)=1$. Similar to Fig. 2 four cases are depicted: (1) no driving ($s=0$), isolated two-level system ($g=0$) (dashed line with filled squares ■), (2) with resonant driving ($s=\Delta_0, \omega_{\mathrm{L}}=\Delta_0$), no dissipation ($g=0$)(dotted line), (3) no driving ($s=0$), with dissipation ($g=0.05\Delta_0, \Gamma=0.1\Delta_0$) (dashed-dotted line with asterisks $*$) and (4) with resonant driving ($s=\Delta_0, \omega_{\mathrm{L}}=\Delta_0$) and with dissipation ($g=0.05\Delta_0, \Gamma=0.1\Delta_0$) (full line). The temperature is chosen to be $T=0$ and and the cavity-mode frequency is set to $\Omega=\Delta_0$.